CEN 133 Hw 4 Bonus Homework

Problem 1 (70 pts). Please first read the "Definition of Integrals. Approximation and Riemann Sums" sections below. You are requested to implement the following procedure:

(riemann-sum function lower-limit upper-limit number-of-rectangles sum-type)

This function should find the riemman sum according to defined parameters:

function: The function which's integral is to be approximated. This parameter should be a procedure.

lower-limit: The lower limit for the integral. This parameter should be an integer variable.

upper-limit: The upper limit for the integral. This parameter should be an integer variable.

number-of-rectangles: The number of rectangles in riemman sum. This parameter should be an integer variable.

sum-type: can be one of left sum, right sum, middle sum. This parameter should be an integer variable.

Please use the generalized sum function:

```
(define (sum term a next b)
 (if (> a b)
    0
    (+ (term a)
      (sum term (next a) next b))))
```

Also please use the lambda abstraction in your homework when you think it is necessary.

Please find the following

Approximate the area under the curve of the function $f(x) = x^4$, by applying the below parameters:

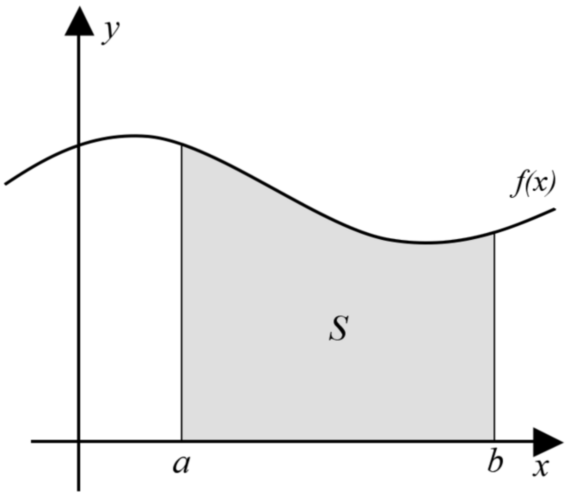
- Lower Limit:0, Upper Limit:8, Number of Rectangles:2, Sum-Type: Left Sum
- Lower Limit:0, Upper Limit:8, Number of Rectangles:4, Sum-Type: Right Sum
- Lower Limit:0, Upper Limit:8, Number of Rectangles:4, Sum-Type: Middle Sum
- Lower Limit:0, Upper Limit:8, Number of Rectangles:8, Sum-Type: Middle Sum

Definition of Integrals. Approximation and Riemann Sums

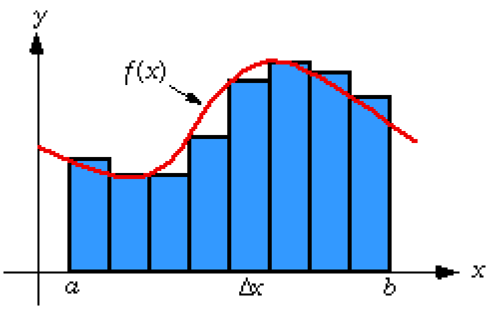
This information is taken from: http://www.msstate.edu/dept/abelc/math/integrals.html

Approximating Areas using Riemann Sums

As you should recall, there are two types of integrals: **indefinite** and **definite**. An indefinite integral does not have bounds, and is merely the mathematical anti-derivative of the function (+C, of course). The nice-and-simple definition of an *definite* integral is the "area under the curve" of a function *f* between the bounds *a* and *b*.



Now let's turn this definition into something a little more mathematical. Let's approximate the area under the curve of *f*, employing the use of *rectangles*. First, we'll chop the interval [*a,b*] into *n* segments, find the area of the rectangles, and sum those areas together.

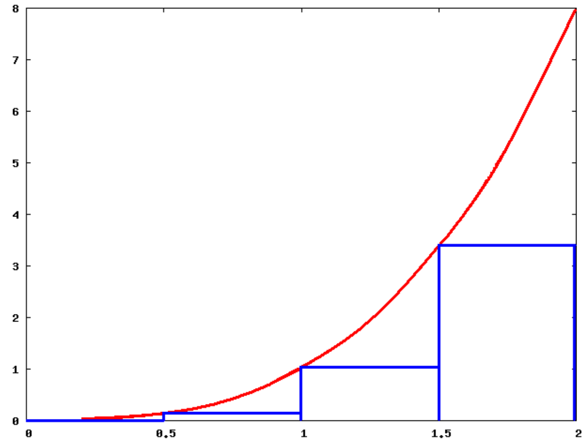


As you can see, the "width" of each rectangle is equal to the segment width ($\Delta x = x_{i+1} - x_i = (b - a)/n$), and the "height" is equal to the value of the function $f(x_i)$. Written out in mathematical terms, we get an equation like this:

$$\text{Area} \approx \sum_{i=0}^{n-1} f(x_i)(x_{i+1} - x_i)$$

This method of integral approximation is known as a **Riemann sum**. There are 3 basic types of Riemann sums:

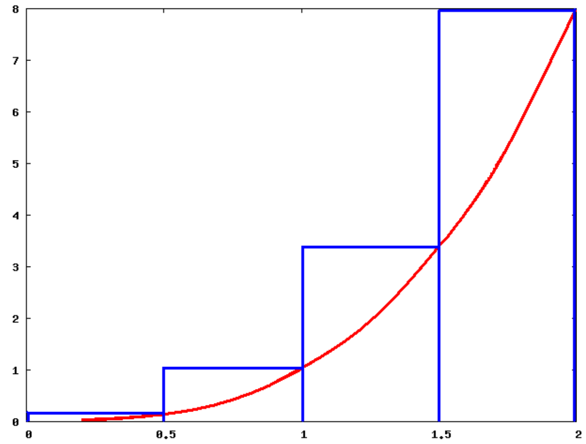
1)



The "left" sum, which measures the heights from where the left point of the interval touches the curve. The equation looks like:

$$\text{Area} \approx \sum_{i=0}^{n-1} f(x_i)(x_{i+1} - x_i)$$

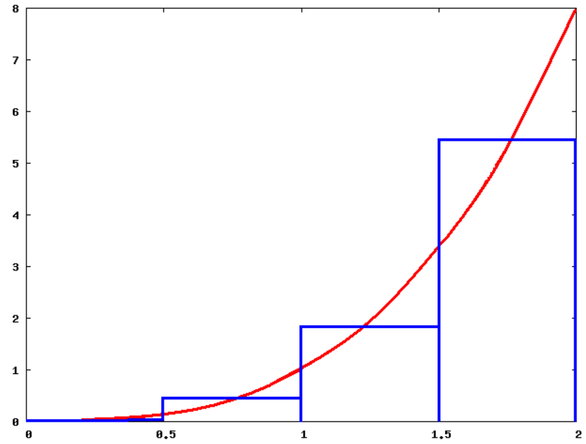
2)



The "right" sum, which measures the heights from where the right point of the interval touches the curve. The equation looks like:

$$\text{Area} \approx \sum_{i=1}^{n} f(x_i)(x_i - x_{i-1})$$

3)



The "middle" sum, which measures the height from halfway between the left and the right points of the interval. The equation looks like:

$$\text{Area} \approx \sum_{i=1}^{n} f\left(\frac{x_i + x_{i-1}}{2}\right)(x_i - x_{i-1})$$

The middle sum is the best choice for approximating an integral, but its important to understand all of these methods.

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Homework Policies:
Cheating is strongly discouraged.

1. Late homeworks will be graded as 0.
2. File naming for the homework:
   Send your file as: CEN133Hw4_“YourStudentNo”.rkt
   example: CEN133Hw4_80201022.rkt
3. When sending your homework the subject part of your e-mail should be CEN133Hw4_ YourStudentNo.

   Note: Please obey these grading policies, unless your grade will be decreased.
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